MATH412

Course Summary

Course : MATH412 **Title :** Graph Theory
Length of Course : 8
Prerequisites : MATH305 **Credit Hours :** 3

Description

Course Description: This course studies set theory, counting techniques such as permutations, combinations, generating functions, partitions and recurrence relations, Polya's theorem, Hamiltonian and Euclidian properties of graphs, matchings, trees, coloring problems and planarity. (Prerequisite: MATH305)

Course Scope:

This course broadens the foundation for mathematicians while improving their skill set for solving technical problems. Its purpose is to provide students with advanced mathematical skills, to promote their deeper knowledge, and to develop them as problem solvers as they face today's challenging issues. Graph Theory is an old area of study that has recently developed into a powerful set of tools. This course is a portion of the Applied Mathematics concentration which is designed to focus on the knowledge of advanced mathematics; the critical thinking skills necessary to assess, evaluate, and apply practical solutions to complex problems; and the application of quantitative and qualitative methods in a wide variety of work disciplines.

Objectives

After successfully completing this course, you will be able to

CO-1 Explain basic definitions and properties associated with simple planar graphs, including isomorphism, connectivity, and Euler's formula.
CO-2 Describe when a graph is a useful mathematical tool to solve problems in mathematics, the sciences and the environment.
CO-3 Describe the difference between Eulerian and Hamiltonian graphs.
CO-4 Use graph theories to solve mathematical problems.
CO-5 Formulate logical and coherent proofs including proofs by contradiction and induction.
CO-6 Analyze studies of topics related to graph theory including combinatorics and computer science.

Outline

Week 1: Introduction to Graph Theory. Definitions and introductory level problems.

Learning Objectives

LO-1: Know the definitions of vertex set, edge set, walk, trial, and path.

LO-2: Identify simple graphs, planar graphs, digraphs, and bipartite graphs.

LO-3: Establish an isomorphism between applicable graphs.

LO-5: Identify connected and disconnected graphs and their complements.

LO-16: Prove basic results for several different types of graphs.

LO-18: Apply a variety of techniques to prove theorems and corollaries.

Readings

Text Readings:

From R. J. Wilson

- Chapter 0
- Chapter 1: § 1.1-1.3

Assignments

Forums:

1) Introduce Yourself

2) Non-isomorphic Graphs with 6 Vertices

Homework:

Problems in Wilson

Required Problems: 1.1, 1.3, 1.4, 1.5, 1.8, 1.13, 1.16, 1.17, 1.22, 1.25, 1.26, 1.27, 1.31, 1.33, 1.35, 1.36, 1.42
Optional Problems: 1.6, 1.7, 1.11, 1.14, 1.18, 1.19, 1.28, 1.30, 1.32, 1.37, 1.38, 1.41

Week 2: The identification and fundamentals of Eulerian and Hamiltonian graphs.

Learning Objectives

LO-1: Know the definitions of vertex set, edge set, walk, trial, and path.

LO-6: Recognize mathematical problems that can be solved using graphs.

LO-9: Identify Eulerian cycles and Eulerian trails.

LO-10: Identify Hamiltonian cycles and Hamiltonian paths.

LO-16: Prove basic results for several different types of graphs.

LO-17: Memorize several fundamental results and their associated proofs.

LO-18: Apply a variety of techniques to prove theorems and corollaries.

Readings

Text Readings:

From R. J. Wilson

- Chapter 2: § 2.1-2.3

Assignments

Forums:

1) Connected digraphs

Homework:

Problems in Wilson

Required Problems: 2.1, 2.3, 2.7, 2.9, 2.12, 2.16, 2.17, 2.24, 2.28, 2.30, 2.31
Optional Problems: 2.2, 2.5, 2.6, 2.13, 2.15, 2.19, 2.21, 2.25(i), 2.27, 2.29, 2.32

Quiz #1

Week 3: Applications of simple graphs. Applications of Eulerian and Hamiltonian walks.

Learning Objectives

LO-6: Recognize mathematical problems that can be solved using graphs.

LO-7: Know areas of scientific study where graphs are often used.

LO-8: Understand the qualities of graphs that make them useful in applications.

LO-11: Use directional graphs to solve problems.

LO-12: Use Eulerian and Hamiltonian graphs to solve problems.

LO-15: Use weighted graphs to solve problems.

Readings

Text Readings:

From R. J. Wilson

- Chapter 1: § 1.4
- Chapter 2: § 2.4

Assignments

Forums:

1) Eulerian and Hamiltonian graphs

Homework:

Problems in Wilson

Required Problems: 1.47, 1.49, 2.36, 2.38, 2.40, 2.41, 2.42
Optional Problems: 1.50, 2.37, 2.39, 2.43

Project #1

Week 4: Introduction to trees. Definitions and applications for spanning trees and counting trees.

Learning Objectives

LO-6: Recognize mathematical problems that can be solved using graphs.

LO-8: Understand the qualities of graphs that make them useful in applications.

LO-13: Use spanning trees and counting trees to solve problems.

LO-15: Use weighted graphs to solve problems.

LO-17: Memorize several fundamental results and their associated proofs.

LO-18: Apply a variety of techniques to prove theorems and corollaries.

LO-21: Understand how combinatorics relates to Graph Theory.

Readings

Text Readings:

From R. J. Wilson

- Chapter 3: § 3.1-3.3

Assignments

Forums:

1) Application of Counting Trees

Homework:

Problems in Wilson

Required Problems: 3.1, 3.2, 3.4, 3.6, 3.7, 3.12, 3.13, 3.15, 3.20, 3.21, 3.22, 3.25, 3.26
Optional Problems: 3.3, 3.5, 3.8, 3.9, 3.17, 3.18, 3.23

Writing Assignment

Week 5: Planar graphs and Euler's formula.

Learning Objectives

LO-2: Identify simple graphs, planar graphs, digraphs, and bipartite graphs.

LO-3: Establish an isomorphism between applicable graphs.

LO-4: Apply Euler's formula for planar graphs.

LO-14: Use planar graphs and Euler's formula to solve problems.

LO-17: Memorize several fundamental results and their associated proofs.

Readings

Text Readings:

From R. J. Wilson

- Chapter 4: § 4.1-4.3

Assignments

Forums:

1) Using Euler's Formula

Homework:

Problems in Wilson

Required Problems: 4.1, 4.2, 4.3, 4.12, 4.13, 4.17, 4.20, 4.22, 4.23, 4.24, 4.29
Optional Problems: 4.4, 4.6, 4.8, 4.15, 4.16, 4.19, 4.25, 4.26, 4.32(i)

Quiz #2

Week 6: Coloring graphs and graphs with faces. The four color problem.

Learning Objectives

LO-6: Recognize mathematical problems that can be solved using graphs.

LO-7: Know areas of scientific study where graphs are often used.

LO-8: Understand the qualities of graphs that make them useful in applications.

LO-12: Use Eulerian and Hamiltonian graphs to solve problems.

LO-17: Memorize several fundamental results and their associated proofs.

LO-18: Apply a variety of techniques to prove theorems and corollaries.

Readings

Text Readings:

From R. J. Wilson

- Chapter 5: § 5.1-5.4

Assignments

Forums:

1) Chromatic Polynomials

Homework:

Problems in Wilson

Required Problems: 5.1, 5.2, 5.3, 5.9, 5.10, 5.11, 5.17, 5.19, 5.21, 5.22
Optional Problems: 5.5, 5.8, 5.13, 5.16, 5.20

Project #2

Week 7: More applications of graphs. Networks as graphs to include applications.

Learning Objectives

LO-6: Recognize mathematical problems that can be solved using graphs.

LO-7: Know areas of scientific study where graphs are often used.

LO-11: Use directional graphs to solve problems.

LO-15: Use weighted graphs to solve problems.

LO-19: Apply graphs to network applications.

LO-20: Apply trees to computer science applications.

Readings

Text Readings:

From R. J. Wilson

- Chapter 6: § 6.1-6.3

Assignments

Forums:

1) Application to networks

Homework:

Problems in Wilson

Required Problems: 6.1, 6.2, 6.3, 6.5, 6.13, 6.14, 6.18, 6.19
Optional Problems: 6.8, 6.11, 6.15, 6.17, 6.20, 6.21

Quiz #3

Week 8: Comprehensive Final Exam

Learning Objectives

LO-1: Know the definitions of vertex set, edge set, walk, trial, and path.

LO-2: Identify simple graphs, planar graphs, digraphs, and bipartite graphs.

LO-3: Establish an isomorphism between applicable graphs.

LO-4: Apply Euler's formula for planar graphs.

LO-5: Identify connected and disconnected graphs and their complements.

LO-6: Recognize mathematical problems that can be solved using graphs.

LO-7: Know areas of scientific study where graphs are often used.

LO-8: Understand the qualities of graphs that make them useful in applications.

LO-9: Identify Eulerian cycles and Eulerian trails.

LO-10: Identify Hamiltonian cycles and Hamiltonian paths.

LO-11: Use directional graphs to solve problems.

LO-12: Use Eulerian and Hamiltonian graphs to solve problems.

LO-13: Use spanning trees and counting trees to solve problems.

LO-14: Use planar graphs and Euler's formula to solve problems.

LO-15: Use weighted graphs to solve problems.

LO-16: Prove basic results for several different types of graphs.

LO-17: Memorize several fundamental results and their associated proofs.

LO-18: Apply a variety of techniques to prove theorems and corollaries.

LO-19: Apply graphs to network applications.

LO-20: Apply trees to computer science applications.

LO-21: Understand how combinatorics relates to Graph Theory.

Readings

N/A

Assignments

Forums:

1) Feedback

Final Exam

Evaluation

Reading Assignments: Reading assignments are provided each week. These assignments flow into the Forum discussions and homework problems. Reading assignments are not graded directly. Your conceptual understanding, ability to solve problems, and ability to synthesize material will be evaluated using quizzes, projects, and a final exam.

Forum and Homework Assignments: Mathematics is not a spectator sport. In order to learn the language of Mathematics, you must be engaged with the material. It is critical that you spend time thinking, considering examples, working problems, and discussing ideas with others. The Forums are evaluated in three areas: quantity of posts, quality of posts, and value of interactions. Initial forum posts are due each week. **Your initial post must be made on or before Wednesday, each week. Homework assignments much be attached to your initial post. You may attach your Homework assignment at any time through Sunday of the week it is due.** Optional homework problems are for your benefit and are not required for submission.
+ The Homework are graded for completeness, correctness, and clarity.

+ The Forums are evaluated in three areas: quantity of posts, quality of posts, and value.

Quantity – The initial post for each Forum includes at least 250 words, and a minimum of two interaction posts are required per Forum using at least 100 words each.

Quality – High quality posts are critical to the development of everyone in the course. The overall quality of your posts is evaluated.

Value – Banal posts such as “Good work” and “Nice conclusion” provide no value to the Forum conversations. The key to the Forums is quality interaction. Superior posts promote a valuable conversation and meaningful interaction.

Forum Note: you cannot score points for the quality and value of a post if you fail to meet the minimum quantity.

Writing Assignment: Written communication is a key piece of modern mathematics. The Forums as well as many of the homework problems ask you to develop an argument and write it clearly. In addition to the Forums and homework, you are required to write two formal proofs as writing assignments. These assignments are evaluated according to their validity, readability, and fluency. The definitions for those concepts are given here.

Validity – Validity corresponds to the validity of your arguments. It addresses the extent to which your method is appropriate, your calculations are correct, and your deductions follow the rules of logic.

Readability – If your written work is not readable it cannot be assessed. Since the ability to communicate Mathematics is a focal point for this class, special attention will be paid to the readability of your work.

Fluency – Mathematics is a concise and precise language, and I wish to enhance your fluency. Therefore, part of every assessment will focus on your ability to incorporate correct, established notation and terminology into your written work.

Quizzes: These are the core assessment tools for the assigned readings and homework. Your work will be graded for correctness, completeness, and clarity.

Projects: As with the writing assignment, the projects provide you with an opportunity to improve your Mathematical communication. For the projects you will need to effectively communicate your ideas in a slideshow presentation that includes an audio track. Using audio files you will explain your work with clarity and precision. This assignment will be evaluated in a manner similar to the writing assignments. However, an additional criterion will be evaluated.

Multimedia – A completely integrated presentation incorporating written work with emphases and accompanying audio. The total effect of the presentation is evaluated.

Final Exam: As with the quizzes, the final exam is a core assessment tools for the assigned readings and homework. The final exam is comprehensive, and your work will be graded for correctness, completeness, and clarity.

Grading:

Name	Grade %
Forums	23.00 %
Honor Pledge	0.96 %
Introduction Forum	0.96 %
Week 1 Forum & Homework	2.88 %
Week 2 Forum & Homework	2.88 %
Week 3 Forum & Homework	2.88 %
Week 4 Forum & Homework	2.88 %
Week 5 Forum & Homework	2.88 %
Week 6 Forum & Homework	2.88 %

Week 7 Forum & Homework	2.88 %
Week 8 Feedback	0.96 %
Writing	30.00 %
Project 1	10.00 %
Project 2	10.00 %
Writing Assignment	10.00 %
Quizzes	25.00 %
Quiz 1	5.00 %
Quiz 2	10.00 %
Quiz 3	10.00 %
Final Exam	22.00 %
Final	22.00 %

Materials

Book Title: Introduction to Graph Theory, 5th Ed - The VitalSource e-book is provided via the APUS Bookstore

Author: Wilson

Publication Info: Pearson

ISBN: 9780273728894

Book Title: You must validate your cart to get access to your VitalSource e-book(s). If needed, instructions are available here - http://apus.libguides.com/bookstore/undergraduate

Author: N/A

Publication Info: N/A

ISBN: N/A

Additional Resource

Graph Theory with Applications
By J.A. Bondy and U.S.R. Murty
5th printing, 1982
Sole Distributer: Elsevier Science Publishing Co, Inc.

Web Sites

In addition to the required course texts, many public domain web sites are useful. Here are a few sites to consider if you want to view topology and modern mathematics from a different perspective. Please abide by the university's academic honesty policy when using Internet sources. Note web site addresses are subject to change.

Site Name **Web Site URL/Address**

Introduction	https://www.youtube.com/watch?v=gbPFyGP_EzU
Eulerian	https://www.youtube.com/watch?v=5M-m62qTR-s
Hamiltonian	https://www.youtube.com/watch?v=AamHZhAmR7o
Another Introduction	https://www.youtube.com/watch?v=HmQR8Xy9DeM
Undirected Graphs	https://www.youtube.com/watch?v=Vfm2h1MZJNM
Isometric Graphs	https://www.youtube.com/watch?v=Xq8o-z1DsUA
Euler's Theorem	https://www.youtube.com/watch?v=VRcX9Fzu1Jo
Spanning Trees	https://www.youtube.com/watch?v=0IjjRM8hWjU
Minimum Spanning Tree	https://www.youtube.com/watch?v=d4BEgzK08JE
Shortest Path	https://www.youtube.com/watch?v=WUMxRp3xei0
Nearest Neighbor	https://www.youtube.com/watch?v=zPgsNsOfxQ8
Dr. Herke Introduction	https://www.youtube.com/watch?v=S1Zwhz-MhCs
Dr. Herke Examples	https://www.youtube.com/watch?v=lLOvB1qFE1E
Dr. Herke Families	https://www.youtube.com/watch?v=71XbdtoG7P8
Dr. Herke Matrices	https://www.youtube.com/watch?v=LUDNz2bljWI

Dr. Herke has an extensive video lecture series on Graph Theory. Only four of her presentations are listed

above. If you want more information, please follow any of her video links and look for more lessons.

Course Guidelines

Citation and Reference Style

- Attention Please: Students will follow the APA Format as the sole citation and reference style used in written work submitted as part of coursework to the University. Assignments completed in a narrative essay or composition format must follow the citation style cited in the APA Format.

Tutoring

- Tutor.com offers online homework help and learning resources by connecting students to certified tutors for one-on-one help. AMU and APU students are eligible for 10 free hours* of tutoring provided by APUS. Tutors are available 24/7 unless otherwise noted. Tutor.com also has a SkillCenter Resource Library offering educational resources, worksheets, videos, websites and career help. Accessing these resources does not count against tutoring hours and is also available 24/7. Please visit the APUS Library and search for 'Tutor' to create an account.

Late Assignments

- Students are expected to submit classroom assignments by the posted due date and to complete the course according to the published class schedule. The due date for each assignment is listed under each Assignment.
- Generally speaking, late work may result in a deduction up to 15% of the grade for each day late, not to exceed 5 days.
- As a working adult I know your time is limited and often out of your control. Faculty may be more flexible if they know ahead of time of any potential late assignments.

Turn It In

- Faculty may require assignments be submitted to Turnitin.com. Turnitin.com will analyze a paper and report instances of potential plagiarism for the student to edit before submitting it for a grade. In some cases professors may require students to use Turnitin.com. This is automatically processed through the Assignments area of the course.

Academic Dishonesty

- Academic Dishonesty incorporates more than plagiarism, which is using the work of others without citation. Academic dishonesty includes any use of content purchased or retrieved from web services such as CourseHero.com. Additionally, allowing your work to be placed on such web services is academic dishonesty, as it is enabling the dishonesty of others. The copy and pasting of content from any web page, without citation as a direct quote, is academic dishonesty. When in doubt, do not copy/paste, and always cite.

Submission Guidelines

- Some assignments may have very specific requirements for formatting (such as font, margins, etc) and submission file type (such as .docx, .pdf, etc) See the assignment instructions for details. In general, standard file types such as those associated with Microsoft Office are preferred, unless otherwise specified.

Disclaimer Statement

- Course content may vary from the outline to meet the needs of this particular group.

Communicating on the Forum

- Forums are the heart of the interaction in this course. The more engaged and lively the exchanges, the more interesting and fun the course will be. Only substantive comments will receive credit. Although there is a final posting time after which the instructor will grade comments, it is not sufficient to wait until the last day to contribute your comments/questions on the forum. The purpose of the forums is to actively participate in an on-going discussion about the assigned content.
- "Substantive" means comments that contribute something new and hopefully important to the discussion. Thus a message that simply says "I agree" is not substantive. A substantive comment contributes a new idea or perspective, a good follow-up question to a point made, offers a response to a question, provides an example or illustration of a key point, points out an inconsistency in an argument, etc.
- As a class, if we run into conflicting view points, we must respect each individual's own opinion. Hateful and hurtful comments towards other individuals, students, groups, peoples, and/or societies will not be tolerated.

University Policies

Student Handbook

- Drop/Withdrawal policy
- Extension Requests
- Academic Probation
- Appeals
- Disability Accommodations

The mission of American Public University System is to provide high quality higher education with emphasis on educating the nation's military and public service communities by offering respected, relevant, accessible, affordable, and student-focused online programs that prepare students for service and leadership in a diverse, global society.